Application of set theory in combination with Karnatic techniques

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1. Introduction

Karnatic music is the music practiced in the province of Karnataka in South India. The origins of this music date back to the time of the "Vedas" (4000-1000 B.C.) and there are even known writings from about two hundred years before the Christian era in which Karnatic techniques are described.[1] Over the centuries, a very diverse set of rhythmic and melodic techniques were developed which were until recently virtually unknown to western musicians. Thanks to the work of amongst others, Rafael Reina at the Conservatory of Amsterdam, these karnatic techniques have been made available to western muscians in an understandable way. It is striking how many of these techniques join seamlessly with contemporary western compositional techniques.

In a general comparison between karnatic music and composed western music, several observations spring to mind: in the rhythmic sphere, it is obvious that karnatic music offers many more opportunities for structure and variation than does its western counterpart. As far as melody is concerned, there are certainly overlaps, for example in the use and development of melodic cells. In the field of harmony, one comes to the surprising conclusion that this aspect of music in Karnatic theory is hardly or not at all considered, while it is arguably the most important subject of development and renewal in the West in the preceding centuries. It is a challenge to discover if the specific qualities of east and west can be united.

In this article, I discuss how Karnatic techniques can be combined with one of the most advanced western harmonic systems, that of "set theory." Set theory has been used by theorists and composers to analyze and develop post-tonal harmony. Before my encounter with Karnatic music, I had considerable experience in this field.[2] During my study of Karnatic music, I wrote several pieces in which I brought karnatic rhythmic principles together with the use of set theory in place of the characteristic microtonal ragas. This article is a report of my findings. The first part deals with certain general observations concerning the combination of different techniques. In the second part I will give a detailed analysis of fragments of my work. Finally, there are several conclusions and suggestions for further development.

Certain expressions from Karnatic and Set theory (including certain notations I have developed myself) will be described in the glossary at the end or in the text itself very

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[1] See among others, "A gentle introduction to South Indian Classical (Carnatic) Music" by Mahadevan Ramesh (1999) in which the "Natyashastra" of Bharata is presented as the first source. This text can be found on the internet via www.khazana.com.

[2] See among others my article in Tijdschrift voor Muziektheorie: "Lux et Veritas: De set-theorie van Allen Forte en toepassingen hiervan voor componisten" (Lux et Veritas: The set-theory of Allen Forte and its uses for composers) (II/2, May 1997) and "Set-theorie, Toonklok en de P-techniek voor componisten" (Set theory, the Toonklok, and P-technique for composers) (VII/1, February 2002)

summarily. Greater understanding and description of these expressions is readily available and is not the subject of this article.

2.	General observations

2.1.	Harmony in Karnatic music

In Karnatic music, vertical harmonies are the result of the melodic (horizontal) motion of voices according to the rules of the particular raga, which determine for example the ascending or descending motion of given notes, as well as general rules of variation and imitation applied to the melodic cells that are used as thematic material. There are no explicit rules about the resulting intervals. In his article "Microtonality in South India: not so modal" (1998) pag. 2, Rafael Reina says: "What about harmony, orchestration or counterpoint? The answer is simple: they don't exist in Indian music."[3]

This absence of rules does not result in a cacophony thanks to the following:

1) the number of melodic voices in karnatic music is limited; there is most often one main voice while other voices accompany in the background, usually mostly imitating the pitch material of the main voice. This contrasts with the rhythmic aspect, in which independent voices (e.g. percussion instruments) create polyrhythmic patterns

2) there are no chordal instruments

3) the music is always governed by a certain raga with limited pitch material for long stretches of time. While it is true that the number of pitches can increase through the use of *gamaka*s (ornaments) and *sruti bhedam* (the use of alternate microtonal pitches within the raga), both techniques obey strict rules of motion which do not allow the use of outside pitches for independent harmonies. Generally speaking, one could say that the use of ragas with 22 possible *sruti*s (microtonal pitches) within an octave makes the use of any harmonic theory problematic enough in itself. The number of different ragas runs in the thousands, as well as the number of possible interval combinations. In addition, each tone within the raga most often fulfills a specific function, with a "personal character," as it were, which is lost in a harmonic approach. Furthermore, "functional harmony" as it exists in Western music is simply not applicable since there is no modulation. In this respect, it could also be mentioned that harmonic theory (apart from the rules of counterpoint) was not really developed in the west until equal temperament became general practice, which made modulations to all keys possible.

Nonetheless, it is possible to decide upon certain chords on a raga-by-raga basis for use within a certain composition; several students of Karnatic music in Amsterdam have experimented with this approach.

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[3] This is confirmed by Mahadevan Ramesh in the aforementioned "A gentle introduction to South Indian Classical (Carnatic) Music," in which he states that "Orchestration and harmony are absent in Indian classical music."

2.2   the application of set theory on ragas

After the dissolution of functional harmony in post-tonal Western music, set theory offered the first successful attempt towards a new harmonic theory whose application reaches beyond the individual techniques of individual composers in a particular piece. Set theory makes it possible to categorize post-tonal harmonies on the basis of their intervallic content and offers techniques to describe the development and variation of chords.

A direct translation of set theory on ragas is impractical; since set theory is based with the *interval-content* of groups of notes, there are an innumerable amount of combinations possible in a raga of 22 srutis, such that it no longer makes sense to categorize them, let alone recognize them by ear.

To give and example: in the "well-tempered" 12 tone system, there are only 12 kinds of triads; in ragas with 22 tones, this number grows to 40, and that without taking into account the variations in distance between different quarter tones in a raga; the actual number of different combinations is many times greater, and we are speaking now only of triads.

The only possibility is not to base the set theory on intervals, but on the numerical order of scale degrees in a raga, thereby coming into the field of an already existing technique in Karnatic music: the transposition and mirroring of groups of notes within a raga by means of *soram transposition* and *tanam*.

In this case however, we would hardly be in the realm of harmonic theory since the harmonies (based on intervallic content) of identical sets can be entirely different. The application of set theory on microtonal ragas thus does not seem particularly fruitful.

2.3   the use of use of Western equal temperament in combination with Karnatic
       rhythmic techniques

In principle, all Karnatic rhythmic techniques, such as the use of *tala*s, *mohara*s, *korvai*s, *tirmana*s, *mukthay*s, polyrhythm and polypulse by means of *gati-jathi* relationships, *anuloma/pratiloma*, *yati*-phrases, *nadai bhedam* and *metrical modulations*,[4] is very much possible in an environment of Western equal temperament. As soon as one does away with the use of ragas and their emphasis on the expressive possibilities of individual notes, harmony necessarily becomes an important expressive and structural tool. Each composer can then use a personal system in which the rhythmic and harmonic aspects will have to fit with and strengthen one another. The harmonies, whether they are tonal, post-tonal, or make use of set theory, can be used to emphasize and clarify the rhythmic structures. On the other hand, phrasing and larger rhythmic structures should be fine-tuned to fit with the structurally designed harmonic progressions.

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[4] Some of these terms are described in the glossary, only if they reoccur in this article.

2.4   technical considerations in the use of set theory in combination with karnatic techniques

The use of set theory works very well towards the goals described above, as there are many possibilities for variation while keeping an underlying basic sonority invariant or precisely in making transitions more to extended harmonies. Moreover, there is an interesting option which I used in *Raaf*, simply to consider sets of 6 or more elements *to be ragas*, and then to apply techniques to them that are used on ragas in Karnatic music. Specific technical problems arise when several Karnatic rhythmic structures are happening simultaneously. Then choices must be made either (1) to choose one of these structures as the determining harmonic factor or (2) to let each structure determine its own harmonic development within its own layer – with somewhat unpredictable results – or (3) to start with a basic harmonic development that is independent of the rhythmic structures but that for example, coincides with the changes of talas.

I tried out several of these possibilities both in *Raaf* and in *Rondo.*

3. Examples of the use of set theory in combination with Karnatic techniques in my own work.

3.1.   *Raaf*[5] for alto saxophone and tape

The tala which is kept throughout the whole piece is 10 beats long, subdivided Lagu 7-Anudrutam (1)-Drutam (2),[6] and notated in 7/4 and 3/4 bars. The accompanying tape is mostly meant to emphasize the tala and to create a harmonic background.

Before beginning to write *Raaf*, I had already been using set theory for years in my writing. Usually my harmonies are based on hexachordal sets. Two of my favorite sets, used among others in *Square Roots* and *La Bonne Chanson* are 6-19 (with prime-form 013478) and 6-5 (with prime form 012367; this was also one of Arnold Schoenberg's favorite sets – used in *Variationen für Orchestra* Opus 31). These were both used as main harmonies in *Raaf*, together with their complementary and secondary harmonies.

3.1.1   the set as raga

I came up with the idea to use 6-19, expanded by 1 extra note (indicated with respect to the prime form with the number 9), as a raga to develop a theme.
Fig. 1 shows the prime form of 6-19 with the added note. Fig. 2 shows the theme with a length of 1 tala, which is developed along Karnatic principles with the use of a cell of (in this case) 4 notes in its original and transposed form. Fig. 3 gives the 7 possible transpositions of the main cell, followed by the set type of the resulting tetrachords. In the theme T^0, T^5 and T^1 are used consecutively;[7] it should be noted that there is always

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[5] *Raaf* (2002) was written for Raaf Hekkema, alto saxophone; first performance took place June 20, 2003.
[6] *lagu, anudrutam* and *drutam* are structural units within a *tala*
[7] the notation T^0 to T^6 here indicates transpositions to other "scale degrees" within the set (comparable to diatonic transposition), not to be confused with the notation T0 to

one common note to act as a pivot between the cells. The last G (in the second beat of the 3/4 bar) does not belong, strictly speaking, to the T1 cell, but is introduced for melodic reasons as a descending alteration of the A directly before it.

Using various six-note sets as a harmonic environment, I kept to the following rules of thumb in chord changes:

1) change of set type is only allowed on the Tala Sam[8]

2) chord changes based on complementary sonorities (the missing six tones from the chromatic spectrum) are allowed on the Tala Sam or Anga Sam.[9] The set-type thus stays the same, or changes to the z-related[10] set with the same intervallic content. I have compared this change from a hexachord to its complement with a "dominant" function. One always stays with a sonority based on the same intervallic content, yet nevertheless a completely new sound is created simply because new notes are used. The "dominant" analogy comes from the idea: "the same thing (in the case of the dominant: the same key), but the opposite, with the tendency to go back to the original."

3) chord changes to the same set type with 2 or more common notes are allowed on every beat

In this way the harmony supports the tala divisions. These rules are meant for developmental passages; I strayed from them in the case of cadences or short, independent fragments without a clear development.

In bar 4, after the first introduction of the theme, there is a harmonic change of the "complementary-type." The seven-tone raga from the beginning contains hexachord 6-44 (with the prime form 0 1 2 5 6 9), in this case in the form of (9 8 7 4 3 0). On the Tala Sam of the second tala, the harpsichord plays the complement of this set (see fig. 4). This complement consisting of (10 11 1 2 5 6) is again another 6-19 set in T10 in relation to the original 6-19 that served as a starting point for the first theme.

The theme is then repeated in *khanda*, i.e. instead of motion in sixteenth notes, the same rhythmic pattern is repeated in quintuplets, retaining the original rhythmic phrasing. In order to resolve on the following Tala Sam, the theme only begins on the third beat. The theme is, along with the harmony, transposed to T10, whereby some notes are heard an octave higher or lower in order to create a completely different contour to the melody. (This technique is common in Karnatic music in the form of *melodic sanghati* and *soram transposition*).

T11 which is used in set theory to denote the number of minor seconds in the case of chromatic (upward) transposition.

[8] Tala Sam indicates the beginning of a new tala

[9] Anga Sam indicates a secondary landing point in the Tala, in this case at the start of the 3/4 bar.

[10] 2 sets with the same intervallic content but a different prime form are z-related. With hexachords, complementary sets always have the same intervallic content, and therefore either have the same prime-form or are z-related.

3.1.2 *gati-bhedam* sequence

After that begins a development in which the harmonic mechanisms described above are applied on various transposition and inversion levels. Subsequently the solo voice and accompaniment go through a so-called *Tree of gati-bhedam* sequence in which different forward-moving speeds or *gatis* (divisions of the beat into 3,4,5 or 7) combined with various systematic accents (*jathis*) follow one upon the other. This Karnatic technique provides certain significant rules to retain continuity. In general, a change of *gati* is prepared with the use of a *jathi* of the same number, and these changes occur on the Tala Sam. Fig. 5 is an exampe of the 4 talas from bar 34 to 41. Fig. 6 provides the schedule of the corresponding *Tree of gati bhedam*.

For the purposes of this article, the main interest is the harmony. This is structured here with the systematic use of 6-5, 6-19 and 6-44 (the complement of 6-19).

The mechanism is as follows (see fig. 7): 6-5 (for the example taken in the form of 0 1 2 3 6 7) is segmented into 4-14 (0 2 3 7) and 2-5 (1 6) p 1.[11] These are in themselves recognizable and characteristic sounds, which together in this p1 position give a very distinctive sonority. Next the 2-5 is transformed by an outward motion of both notes by a distance of a major second: 1 becomes 11 and 6 becomes 8. The result is 6-19, in this example (3 2 0 11 8 7). This 6-10 changes on the Anga Sam (thus on the 3/4 bar) to its complement 6-44, in this case (4 5 6 9 10 1). On the following Tala Sam, the mechanism is repeated on the T4 level. After two repetitions of T4, another T4 would lead to the initial harmony; instead, T2 is taken as a new starting point.

In terms of the melody, generally only notes from the governing sets are used, mostly in groups (cells) that are repeated to emphasize the rhythmic accents of the *gati bhedam* sequence. In some cases, notes not belonging to the governing set can be used; keeping to the analogy of sets as ragas, this can be seen as a *sruti bhedam* in the sense that alternative pitches are used in the place of their regular counterparts. In some cases, it is not a question of alternative notes but rather expressive additions with the character of a suspension or anticipation. The 2-5 and its transformation can be seen in the melody, such as in bar 34 in the 5:4 bracket, or in the accompaniment and in quick alteration, such as in bar 36.

If we continue to use sets as raga, then the transformation of 2-5 to two adjacent notes also exhibits the phenomenon of *sruti bhedam*. In *sruti bhedam* a note of the raga is temporarily replaced by another pitch. A direct progression of two alternate srutis in one voice or vertically in different voices is not allowed in Karnatic music (comparable to *querstand* in tonal harmony). During set transformation, I would also avoid vertical concurrence, but the consecutive horizontal succession of alternate notes is not disturbing here in my opinion, and sometimes even desirable to emphasize the transformation. A big difference with *sruti bhedam* is naturally that

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[11] 4-14 and 2-5 are set-types; in brackets are the actual note representations. "p1" implies that the 2-5 set is in a position of a minor second away from the starting point of the 4-14 set. For an explanation of set segmentation and corresponding "P-notation" for the underlying position of the segments, see the article "Set theory, the Tone clock and P-technique for composers" (*Tijdschrift voor Muziektheorie* VII/I, February 2002) as well as "P-notation" in the glossary.

sruti bhedam always looks for the smallest possible microtonal interval with relation to the fundamental to which the ear has had time to become accustomed. In the case of sets, it is a question of shifts of a minor or major second in relation to changing representations of a set where absolutely *no fundamental is present*. Consecutive repetitions of the transformed notes can therefore be particularly clarifying.

3.1.3 tirmanas (1)

A *tirmana* follows from bar 44 to 53, i.e. a melodic motive of (in this case) 5 notes that are repeated in different note lengths (see Fig. 8). The note lengths can be systematically decreased (*krama)*, increased (*viloma*) or changed randomly (*vakra*). The underlying harmony once again mainly uses 6-5, 6-19 and 6-44 in various transpositions and inversions. What is primarily of interest here is the relationship of the *tirmana* to these harmonies and how they influence one another; a few details follow here.

In bar 44, the 6-44 from bar 43 is heard first, followed (in the second beat) by the z-related 6-19 in the order (from bottom to top) 7 8 10 11 2 3 with a doubling of 8 and 11. The *tirmana* melody chooses 5 notes from the "raga" belonging to this 6-19, following the example of bar 1. The F in bar 65 doesn't belong to this raga, but can be taken as a *sruti bhedam* with the goal of calling more attention to the second set of 5 notes, now heard in shorter note values. In bar 46, the underlying harmony remains unchanged (contrary to all preceding Tala Sams) so as not to distract from the further development of the first *krama* form. Then other notes of the raga are chosen reinforcing the cyclic character by keeping the *contour* of the 5-note motif unchanged. The chord change to the complementary 6-44 set (6 5 4 1 0 9) in bar 47 coincides with the beginning of the *viloma* form. The start of the retrograde (now lengthening) note lengths correspond to the retrograde contour of the motif. The end of the *viloma* falls in the middle of bar 50 on the second half of the fourth beat. The start of the directly following *vakra* form is emphasized by the transition to another set in the middle of the bar: 6-5 in its known segmentation, in this case 4-14 (4 6 7 11) in the left hand and 2-5 (5 10) in the right in p1 position. The number of common tones is here kept to a minimum for contrast purposes: only 2 (4 and 10). The *vakra* character is further strengthened by randomly varying the contour of the melody. The sequence ends with a short *mukthay* (three-part structure) starting in bar 52, of 4 beats with an internal four-part contour which (as a shortening of the preceding five-part contour) fits the acceleration.

One harmonic detail remains: in bar 52 there is again a case of 6-5 in 4-14 (8 6 5 1)/ 2-5 (7 2) segmentation. On the fifth beat I apply a technique that can be used with all symmetrical sets, thus also here with the 2-5 since each set of two notes is symmetrical (meaning that it is possible to mirror the set in relation to a subsequently determined axis of symmetry—by which the set remains unchanged). The position of 4-14 in relation to the 2-5 could be described as a p 11 with respect to the starting point 7, measured in descending order because both sets unfold in descending direction; the same p 11 in relation to the 2-5 set but then with 2 as a starting point in an upwards motion results in another 4-14 set, namely (1 3 4 8) which together with the 2-5 (2 7) forms another 6-5!
This principle can be described as: the change of note-material while keeping the set type and one subset invariable.

3.2.   *Rondo* for string orchestra and percussion.

In *Rondo* (2003, 17 minutes, commissioned work for the Amsterdam Sinfonietta) I used a number of advanced Karnatic techniques, again in combination with set theory. The piece has a five-part A-B-A-B-A form. In the A parts a *jathi bhedam* sequence is first presented and then developed. A *jathi bhedam* sequence is a pre-determined succession of rhythmic lengths, which together span the length of one *tala*, and that can be used in its entirety or in parts as thematic material for further development.

3.2.1   Hexachords as a harmonic frame for a *jathi bhedam* sequence

After the tempo is set down in the first three bars (see Fig. 9), a strong entrance of the strings in bar 4 introduces the chord progression that will later grow into a complete *jathi bhedam* sequence. At the same time a *metrical modulation* occurs: the systematic accent of 4 x 5 sixteenth notes in bar 4 becomes the new pulse in bar 5. *Metrical modulation* will be further discussed in paragraph 3.2.3.

The chords in bar 4 are six-part and all from different set-types (to be precise: 6-z6, 6-44, 5-28, 6-16, 6-2, 6-8 and 5-10 where doubling occurs in 5-part sets). The chords are purposefully of various types and segmentations, so that they can be recognizable and differentiated as a structural basis for the later *jathi bhedam* sequence. For that reason they are used mostly in the same position in the development; there are, however, some sporadic *sruti bhedam*-like alterations of one or up to two notes of the chord.

In bar 71 (see Fig. 10) the motive from bar 4 has grown into a full *jathi bhedam* sequence: by adding and inserting hexachords and lengthening or shortening the length of each chord a sequence of various chords is created that together fills the entire 13-beat tala. The length of the chords in the progression is as follows (in sixteenth-note values): 3,4,6,4 – 5,2,5,6 – 3,7,5,2 (see Fig. 11). Percussion 3 plays only the accents. The progression, which can be divided into 3 groups of 4 as shown, now can be used thematically. Many things can be done with it. An important addition to the Karnatic technique is that not only the rhythmic sequence but also the succession of hexachords can be used as material. The rhythmic and harmonic sequence can be used simultaneously or completely separate from each other.

An example:

In bars 78 and 79 (together one tala of 13 beats, see Fig. 12), the celli play a version of the rhythmic sequence in *tisra* (2nd speed), i.e. in sextuplet motion. We then have 13 x 6 = 78 matras in the tala, while the original sequence only has 52 matras. To fill the whole tala, gaps of two whole beats + 1 matra of rest are inserted between the three cells a (3,4,6,4), b (5,2,5,6), and c (3,7,5,2) (NB. on the first matra of "rest," a non-accented A# is played. This is for melodic reasons and at the same time to show the end of the long G, whose length would otherwise be unclear).

The violins play a phrase in three parts above this that consists of 1) the beginning of cell b, followed by a 32nd note motive that is introduced shortly before; 2) the complete cell c (3 7 5 2) with a lengthening; 3) (in bar 79) a *mukthay* over 4 beats. A *mukthay* is a structure repeated three times that resolves on the following Tala Sam.

This repeating structure can be long and complex or, as here, very short: 3 x 6 matras in quintuplet motion with 1 matra in between. In the next tala (measures 80-81) the celli and violas play the entire *jathi bhedam* sequence in a (rhythmic) palindrome where the last unit in bar 79, thus the rhythmic length "2" (filled as A#-G#), functions as a pivot. The sequence is then heard in reverse order where the missing "2" are compensated with 2 matras of rest between cell c and cell b. I found this better than to use a complete retrograde beginning with the "2" in bar 80. Such ad-hoc decisions based on esthetic considerations are not inappropriate, on the contrary, they can have great musical value.

The chords do follow the original order, but they are in a T2 transposition with respect to the original. The celli and violas only give the bottom three voices of the chords; above that the violins are rhythmically independent in their own 32nd-note pattern. From bars 2 to 7 this is for example *khanda jathi* 6, i.e. five times a run of 6 matras in quintuplets = 30 matras = 6 beats. Harmonically, the violins follow the chord progression given by the celli and violas, i.e. they always use exclusively (and as completely as possible) the notes of the top half of the chords. Fig. 13 gives an overview of the chords in T2 and their appropriate lengths.

To sum up, from bar 71 to 81 we have seen an intricate web of rhythmic patterns in various speeds that always refer back to the original *jathi bhedam* sequence in which the harmonies are directly derived from the original *jathi bhedam* chords (and thus also the chords in bar 4). For the harmony I have always taken one layer as a starting point, namely that layer whose character is most structural (because it refers to the thematic parts of the composition, and stretches over a longer period of time). The other layers can be rhythmically independent but adapt themselves harmonically.

3.2.2   combining *tirmanas* with harmonic progressions based on set theory

A *tirmana* are also used in *Rondo*, this time consisting of *krama* followed by *viloma*. Unlike the tirmana in *Raaf*, this time it is a melody but a sequence of chords that first shrinks and then expands again. In the *krama* version, 6 chords are repeated by the percussion and pizzicato strings at distances of 10,9, 7, 5, 3, 2 and 1 sixteenth-note, followed by an acceleration into sextuplets and 32nd-sextuplets. Then follows the *viloma* with a sequence this time of 5 chords with continually longer distances: first as 32nd-sextuplets, then in 32nd-quintuplets, regular 32nds, quintuplets, sixteenth-notes, and then distances of 2, 3, 5, 7, 9, and 11 sixteenths. In total, the calculations fill up 7 talas.

Our main interest here is the harmony which is derived from the *jathi bhedam* chords. Fig. 14 gives the six chords in *krama*. I have deviated as little as possible from the chords in bar 71. Only a few times have I changed a note from the original chord, such as for example the low D in the first chord that originally was a C#. This can be seen in the Karnatic spirit again as *sruti bhedam*, an expressive addition or alteration of the chord, temporarily or permanently; the relationship with the original chord still remains clear.

The celli answer each "chord-striking" with a 32nd-note run that uses notes from the chord and occasional passing notes. The violins play in rhythmic counterpoint figures in *khanda* (quintuplets) motion with various *jathis* (systematic accents), using notes

exclusively from the chords given by the tirmana, sometimes anticipating the next chord. Fig. 15 illustrates a fragment in the middle of the *krama* form in which the percussion, violas and double basses play the last three chords from the tirmana sequence with a distance of 9 sixteenth notes and then proceed to the first three with a distance of 7 sixteenths.

3.2.3   metrical modulation

Metrical modulations occur often in Karnatic music; in fact it is the only way to change to a new tempo. There are no metronome markings used, but equivalencies are indicated between note values in the old and the new tempo, in which the note lengths can be embedded in a "tuplet."

The same thing is not unusual in contemporary Western music, but preferably conductors and performers would also like to see metronome markings. To make this easier, even in complex situations, I have developed a simple formula to calculate the new metronome marking (MM2) based on the old (MM1):

If on the left and right of the "=" sign there are simple note values (n1 and n2), then MM2=n2/n1 x MM1.

If the note on the *left* there is embedded in a "tuplet," then the above result must be multiplied by the ratio described by the tuplet: b1/f1 or the "bracket" divided by the"frame" (e.g. for a sixteenth-note quintuplet 5:4).

If on the *right* there is a tuplet, then the result must be multiplied by f2/b2, or the "frame" divided by the "bracket."

Finally, it's possible that MM1 is described in different note values than MM2. In that case the result must be multiplied by N1/N2 or the relationship between the note values left and right.

Together this makes up the formula:

MM2=n2/n1 x b1/f1 x f2/b2 x N1/N2 x MM1

In most cases one or several factors in the formula are equal to 1 because note values left and right are equal or tuplets are absent.

3.2.4   notation problems in the use of Karnatic techniques in a score for Western ensembles

The study of Karnatic music produces a large number of rhythmic techniques that can be used in contemporary Western music compositions. Some techniques correspond directly with the practices of specialized performers of contemporary music (this is true also of the use of quarter tones).

For several years at the Conservatory of Amsterdam, students have been trained in complex Karnatic techniques, and this seems to be very helpful in the performance of other, non-Karnatic contemporary music. For me it is also a challenge as a composer

to use Karnatic techniques in works for performers who have not had specialized training in Karnatic music. One then has to expect great dedication from the performers, who are sometimes treading unknown ground.

The composer cannot expect that the notation used in the Karnatic studies in Amsterdam can be used without greater revisions. It should be noted that notation-methods themselves are also a compromise, since Karnatic music does not use western notation.

Two points that ask for special attention are:

1) the frequent use of systematic accents in a regular or irregular pattern in various *gatis* (subdivisions of the beat in 3, 4, 5, 6, 7 etc.)
2) the use of talas

To begin with the first: an example of systematic accents in a regular pattern is bar 5 of *Rondo* (Fig. 9). The notation used in the Amsterdam training is given in Fig. 16. This notation makes the 4:5 relationship clearer, and becomes especially relevant when there are not quintuplets but triplets, quadruplets or septuplets under the 4:5 bracket, which is not a problem for performers of Karnatic music. For a classically trained orchestra such as the Amsterdam Sinfonietta, with limited rehearsal time, I chose more conventional notation.

Notation problems also occur when dealing with fragments with irregular rhythmic patterns, such as the whole *jathi bhedam* sequence from bar 71 (Fig. 10). According to "Karnatic notation" the rhythmic units would be emphasized by placing them under one beam, as shown in Fig. 17 in the violin. The advantage in this is that the rhythmic units are clearly distinguished separately and therefore performed independent of the beat, without a sort of "syncopation" feel. This becomes all the more relevant when the rhythmical sequens has thematic importance, as in a jathi-bhedam sequence.[12]

However, notation such as that in Fig. 17 would lead to great confusion in a classical orchestra. The option to have the conductor also conduct the *jathi bhedam* sequence is in this case negated by the independent parts of the xylophone and vibraphone. Also in the following development after bar 71 I used conventional notation in the *jathi bhedam*, such that rhythmic units are only denoted by accents; I did however, place a reminder in the foreword to the score and parts that all accents in the piece refer to thematic rhythmic units, and must therefore be strongly emphasized and are more important than the downbeats.

Considering the phenomenon of *tala*: this is a structural unit whose length lies somewhere between the Western concept of bar, phrase, or even section, and can be a good addition to the composer's repertoire. It makes the rhythmic structures such as those described above possible or more interesting. Very long talas or tala-parts can create problems in terms of notation. A tala of 13/4 such as in the second part of

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[12] The notation of rhythmical units under one beaming is similar to the "first" (and preferred) notation described in "The Technique of My Musical Langauage" by Olivier Messiaen, Chapter VII.

Rondo will always be divided into shorter parts by conductors and performers. To save time, the composer had better do this beforehand. In order not to lose the important cyclic character in performance, a good option is to use dashed barlines. The subdivision can then be fitted to the phrasing to have a musical and not arbitrary meaning, without making the longer units invisible.

4.   **Conclusions**

Set theory and Karnatic rhythmic techniques can be united quite successfully. One could even say that they are, in their respective fields of harmony and rhythm, comparably systematic approaches based on number ratios. The examples of combined use in this article are only the beginning of an extensive field of possibilities that can be used in compositions. Finally, it is important to mention that with little experience of Karnatic techniques and/or set theory, analyses such as part 3 of this article may give the impression that complexity is an end in itself, in which music is placed in a tight straight-jacket. The opposite is true: musical expression comes first. The fundamental structures of set theory and Karnatica leaves plenty of room for this because they can be filled up in many ways.

Of course one should also never be hesitant to stray from the rules for urgent musical reasons; the many places where I have done so in the analyzed pieces were less relevant for the purposes of this article.

Glossary

Some terms relating to Karnatic music:

gati: subdivision of the beat into 3,4,5 or 7 parts (*matras*), called *tisra, chatursra, khanda* and *misra* respectively. These can be further doubled or halved.

jathi: a systematic accent on the 3rd, 4th, 5th, 6th or 7th *matra* whereby a new pulse is created within a *gati*

jathi bhedam sequence: see 3.2

matra: the smallest note-length within a *gati*

mukthay: a rhythmic structure which is repeated three times (with or without variation) which closes a musical section. Can be very short (several *matras*) or very long (several *talas*)

raga: a collection of 4 to 12 pitches, chosen from the 22 microtonal pitches recognized within the octave in Karnatic music, with their associated rules of motion.

soram transposition: transposition of melodic material to other degrees (*sorams*) of the *raga*, comparable to diatonic transposition. Great variation in the microtonal intervals of motives is created in this way.

sruti: a microtonal pitch

sruti bhedam: replacement of a pitch from the raga by a neighboring *sruti*, following the rules of the chosen *raga* (comparable to alteration).

tala: a rhythmic cycle between 3 to ~30 beats that is held for a considerable length of time (sometimes for the whole piece).

tala sam: the starting point of each new *tala*

anga sam: a secondary downbeat within the *tala*

tanam: permutation of the order within a cell of 3, 4 or 5 notes.

tirmana: melodic motive of 3 to 7 notes that is repeated in different note-lengths. These can be shortened systematically (*krama*), lengthened (*viloma*), or randomly changed (*vakra*).

Some terms relating to set theory:[13]

number representation: in set theory, note names C-C#-D...up to B are represented by the numbers 0,1,2...up to 11 (see explanatory chart Fig. 18).

complement: the collection of pitches from the 12-tone spectrum that are *not* in a particular set.

interval content: the collection of all the interval possibilities available in a set. These are represented using the "interval vector:" intervals are brought back to their smallest ambitus by reducing octavation and then described with six numbers according to the number of minor and major seconds, minor and major thirds, fourths and tritones.

P-notation: the relative position of two sets can be described in "P-notation:" this consists of the set names followed by a lower case p and the number of minor seconds between the beginning point of the normal order of both sets, measured in the direction in which the first set's normal order unfolds. If the

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[13] For further explanation, see a.o. my aforementioned article "Lux et Veritas: The set theory of Allen Forte and its applications for composers." (*Tijdschrift voor Muziektheorie* II/2, May 1997).

second set unfolds in the opposite direction, an (i) is inserted after the p. This notation has been developed by myself for compositional purposes and was first described in the article "Lux et Veritas: The set theory of Allen Forte and its applications for composers" (see footnote 12).

prime form: the position of a set whereby it retains the smallest ambitus, transposed to any starting point indicated by 0. A limited number of prime forms exist. In a complete list of prime forms, each one is represented with the sum of the numbers in that set, followed by its number order.

set: a collection of pitches

Abstract

Karnatic music (the classical music of South India) uses complex rhythmic techniques which can be adapted by contemporary Western composed music. The author discusses the integration of Karnatic techniques with one of the most advanced harmonic systems in the West: set theory, using examples from his own works.